Nicolas Pain (01/2010)

The Epistemological Argument against Mathematical Platonism

Generic formulation of the epistemological argument against Mathematical Platonism :

Benacerraf, Paul, 1973, « Mathematical Truth », Journal of Philosophy 70 : 661-80. Reprinted in : Benacerraf, Paul & Putnam, Hilary (eds.), 1983, *Philosophy of Mathematics : Selected Readings*, Cambridge : Cambridge University Press, pp. 403-420.

See also :

Balaguer, Mark, 1998, *Platonism and Antiplatonism in Mathematics*, Oxford : Oxford University Press.

Colyvan, Mark, 2001, *The Indispensability of Mathematics*, Oxford : Oxford University Press.

Field, Hartry, 1991, *Realism, Mathematics, and Modality*, Oxford : Blackwell.

Gödel, Kurt, 1964, "What is Cantor's Continuum Hypothesis?". Reprinted in Gödel, Kurt, 1995, *Collected Works, Vol II : Publications 1938-1974*, Oxford : Oxford University Press, pp. 254-270 ; and in Benacerraf, Paul & Putnam, Hilary (eds.), 1983, *Philosophy of Mathematics : Selected Readings*, Cambridge : Cambridge University Press, pp. 470-85.

Gödel, Kurt, « The present situation in the foundations of Mathematics ». Lecture given at the *American Mathematical Society* in Cambridge Massachussetts, 29-30 December 1933. Reprinted in Gödel, Kurt, 1995, *Collected Works, Vol. III : Unpublished Essays and Lectures*, Oxford : Oxford University Press, pp. 45-53.

Kitcher, Philip, 1984, *The Nature of Mathematical Knowledge*, Oxford : Oxford University Press.

Maddy, Penelope, 1990, *Realism in Mathematics*, Oxford: Clarendon.

The search for the foundations of Mathematics, as Gödel put it, is divided into a mathematical task, which consists in the axiomatization (to reduce the methods of proof to a short set of rules of inferences which defines the reliable logical inferences ; to define the propositions from which all others derived) of mathematical theories and in the proof that the resulting axiomatized systems are semantically and syntactically complete, decidable and consistent ; and a philosophical task, which purpose is to justify the truth of the axiomatized systems by showing that the axioms are *grounded*, that is to say are an adequate description of objects. Mathematical Platonism is the claim that 1) there exists non-spatio-temporal objects ; and that 2) these abstract objects are the referents of certain mathematical propositions, the

true and meaningful mathematical propositions, those which are adequate statements about abstract objects. By claiming also that we have, by a way or another, an access to abstract objects, notably by a specific cognitive faculty to perceive them, Mathematical Platonism is a way to achieve the philosophical task.

The skeptical argument against Mathematical Platonism addresses this epistemogical capacity. This argument is generally proponed both by nominalists (those who claim that there are only empirical objects and that the only meaningful and true propositions are those which describe empirical objects) and by realists who argue that there are other ways than crude platonism to justify mathematical evidence (the fact that we know for sure that certain mathematical propositions, e.g. « 3 is a prime », are true) with abstract objects. The key point of the classical argument, which can be found in Benacerraf's « Mathematical Truth », is the causation theory of knowledge : A person X knows that p iff X is, in an appropriate way, causally related to the fact that p. However this epistemological argument against Mathematical Platonism can be reformulated without the causation theory of knowledge. We start by showing the claim that human beings can only obtain the knowledge of spatio-temporal entities. Then, we argue that, *ab absurdo*, if Mathematical Platonism is true, then human beings cannot obtain any mathematical knowledge. And since we do have mathematical knowledge, Mathematical Platonism is not the appropriate way to justify the truth of mathematical propositions and our mathematical knowledge.

Paul Bennacerraf :

« My second condition on an over-all view presupposes that we have mathematical knowledge and that such knowledge is no less knowledge for being mathematical. Since our knowledge is of truths, or can be so construed, an account of mathematical truth, to be acceptable, must be consistent with the possibility of having mathematical knowledge : the conditions of truths of mathematical propositions cannot make it impossible for us to know that they are satisfied. (…) [If *X* knows that *p*, it] must be possible to establish an appropriate sort of connection between the truth conditions of p (as given by an adequate truth definition for the language in which p is expressed) and the grounds on which p is said to be known, at least for propositions that one must come to know –that are not innate. In the absence of this no connection has been established between having those grounds and believing a proposition which is true. (…) If, for example, numbers are the kinds of entities they are normally taken to be [that is to say, abstract objects], then the connection between the truth conditions for the statements of number theory and any relevant events connected with the people who are

supposed to have mathematical knowledge cannot be made out. (…) This second condition on an account of mathematical truth will not be satisfied, because we have no account of how we know that the truth conditions for mathematical propositions obtain. »

P1) If spatio-temporal beings can obtain any knowledge, it can only be the knowledge of spatio-temporal objects.

P2) Human beings are spatio-temporal entities.

C1) Therefore human beings can only obtain the knowledge of spatio-temporal objects. (Modus Ponens, P1, P2.)

P3) If any abstract object is a non spatio-temporal entity, then the knowledge of such objects is inaccessible to spatio-temporal beings.

P4) Human beings are spatio-temporal beings.

C2) Therefore, human beings cannot obtain the knowledge of abstract objects. (Modus Ponens, P3, P4.)

P5) If Mathematical Platonism is true, then beings without the knowledge of abstract objects may not have mathematical knowledge.

P6) Human beings do not possess the capacity to know abstract objects. (Intr. C2.)

C3) Therefore, human beings may not have mathematical knowledge. (Modus Ponens, P5, P6.)

P7) Either human beings do possess mathematical knowledge and Mathematical Platonism is not the appropriate way to explain human mathematical knowledge and the truth of mathematical propositions ; either human beings do nos possess any mathematical knowledge and Mathematical Platonism is the appropriate way.

P8) Human beings possess mathematical knowledge.

C4) Therefore, Mathematical Platonism is not the appropriate way to explain human mathematical knowledge and the truth of mathematical propositions. (Modus ponendo tollens, P7, P8.)